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## FLOW OF EMOTIONAL MESSAGES IN ARTIFICIAL SOCIAL NETWORKS

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Models of message flows in an artificial group of users communicating via the Internet are introduced and investigated using numerical simulations. We assumed that messages possess an emotional character with a positive valence and that the willingness to send the next affective message to a given person increases with the number of messages received from this person. As a result, the weights of links between group members evolve over time. Memory effects are introduced, taking into account that the preferential selection of message receivers depends on the communication intensity during the recent period only. We also model the phenomenon of secondary social sharing when the reception of an emotional e-mail triggers the distribution of several emotional e-mails to other people.

*Keywords:* weighted networks, emotions, sociophysics

### 1. Introduction

The World Wide Web (WWW) is the location of various *human actions* that are of interest to many physicists because of the plethora of available data <sup>1,2,3,4,5,6,7,8,9,10</sup> and the complexity of phenomena taking place in techno-social networks. An example is the bursty nature of human activities in cyberspace (e-mails, web-browsing) considered by Barabási <sup>11,12</sup> to be a consequence of decision-based queuing processes. Since activity patterns in e-communities, e.g. social groups emerging due to interactions on the Internet <sup>16,17,18</sup>, are now better understood <sup>13,14,15</sup>, one can ponder more specific issues related to interactions between users in the e-world. Until now, there have only been limited results regarding the influence

of emotions on the structures of e-communities. This issue is specific in the sense that communication via the Internet is different than meetings in the real world. However, emotions are also expressed in e-mails and even more in anonymous comments on blogs or in Internet discussions. What are emotions? There is no agreement among psychologists about a common definition of this phenomenon. Here, we shall understand it as follows: "Emotions are caused by information processing, so called appraisals, that relates internal or external events to personal relevance and implications, taking into account whether the individual can cope with these and how they relate to personal and social norms"<sup>19</sup>. An external event for a specific agent in cyberspace can be as basic as a message containing information that evokes an emotion because of the receiver's personal connection to that message. It is possible to measure the emotions of individual users using a specific equipment<sup>20,21,22</sup>. One can also efficiently detect emotional content in a text<sup>23,24,25</sup> employing the methods of machine learning.

To describe social relationships one can use network-based models. In the first approximation, the presence of any social interaction is shown as a link between two group members. Such networks evolve over time as new group members join and new social interactions occur<sup>27,26</sup>. Evolving *weighted* networks are a natural extension of unweighted ones<sup>34,35</sup>. Weight expresses the strength of a social link that can be measured, for example, by the frequency of existing mutual contacts<sup>30</sup>. It is worth stressing that weight distributions in social networks<sup>34</sup> do not always follow a power law, as might be expected from the model developed by Barrat *et al.*<sup>28</sup>.

Several models of weighted social network have been studied in the literature. Results of the model by Kumpula *et al.*<sup>35</sup>, which assumes a fixed number of users and evolving values of weights between them, are in agreement with data about community structures in telephone networks. Another example of successful modelling of social interactions is the paper by Singer *et al.*<sup>30</sup>, where the authors define a possible friendship as a function of the total number of contacts between the agents and obtain a degree distribution that is in agreement with the data collected from social studies about friendship networks in schools. Although the outcomes from both models are consistent with collected data, none have dealt with weight distributions in social networks.

We believe that link weights are crucial for communities in cyberspace since it is easy to send a single e-mail to a person and only observations of frequent e-mail exchange reveal the significance of real social relations. Their dynamics are obviously driven by affective phenomena that are introduced in our approach in a simplified way.

## 2. Network Structure

We construct an evolving, directed, weighted network of agents in an artificial community where weight  $w_{ij}(t)$  is the number of messages that person  $j$  has already

received from person  $i$  up to the time moment  $t$ . Generally there is  $w_{ij}(t) \neq w_{ji}(t)$  what means that a link from  $i$  to  $j$  can have a different weight than a link from  $j$  to  $i$ . Initially, we start with a fully connected network of  $N$  agents. The number of agents is fixed in time. We shall try to model flows of *positive* emotional messages among the agents; however, our model can be generalized to cases where both positive and negative emotional messages are communicated. The initial condition is a fully connected network of agents where every link possesses the same weight,  $w_{ij}(0) = 1$ . This means that at the very beginning, each group member sent a polite message, such a *Hello Partner*, to every other group member; thus, the initial weight probability distribution is  $P(w) = \delta_{w,1}$ . During the evolution process weights become heterogeneous.

### 3. Models of emotional message transfer in a group

To model a process of emotional message transfer in our e-community, we use several variants of the updating rules, starting with the simplest and most trivial cases and then moving to more complex and more realistic solutions. Our main goal is to determine the most important characteristics of this toy model, which will eventually allow us to look for similarities to real-world data in the future.

#### 3.1. Model 0

Model 0 is a trivial case where in every time step we randomly choose a sender,  $i$ , of an emotional message and then randomly find the recipient,  $j$ . This processes an increase in the number of transferred messages between nodes  $i$  and  $j$  so that  $w_{ij}(t+1) = w_{ij}(t) + 1$ . In such an approach, the fact that messages contain emotional content that can influence agents' actions has no impact. This simple updating procedure gives the Poisson distribution of weights  $P(w) = \frac{\lambda^w e^{-\lambda}}{w!}$ , where  $\lambda = Tp, T$  is the simulation time, and  $p = \frac{1}{N(N-1)}$ .

#### 3.2. Model I – with infinite memory

Unlike the previously described version, **Model I** is equipped with a memory of emotions that were communicated between agents. We randomly find the sender (agent  $i$ ) and then we choose a recipient for his emotional message (agent  $j$ ) using a preferential rule. The agent decides to whom he wants to send a message, taking into account the complete history of communications with other agents. The selection probability is proportional to the number of past messages transferred between nodes  $j$  and  $i$ . The reason for this rule is that the agent  $i$  makes a stronger (emotional) contact with the group members who have already sent him many messages. Here, we stress the fact that, for simplicity, all messages express only positive emotions. In the continuous time approximation, our rule of communications will lead

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to the following equation for changes of the weights  $w_{ij}(t)$ :

$$\frac{\partial w_{ij}(t)}{\partial t} = \frac{w_{ji}(t)}{s_i^{in}(t)} \frac{1}{N}. \quad (1)$$

Here,  $s_i^{in}(t)$  is the temporal incoming strength of node  $i$  defined as  $s_i^{in}(t) = \sum_j w_{ji}(t)$ . Numerical results show that with good agreement we can say  $w_{ij} \approx w_{ji}$  and

$$\frac{\partial w_{ij}(t)}{\partial t} = \frac{w_{ij}(t)}{s_i^{out}(t)} \frac{1}{N} \quad (2)$$

where  $s_i^{out} = \sum_j w_{ij}$  can be estimated as  $s_i^{out}(t) \approx \langle s^{out} \rangle(t)$ . However, the mean value of node strength is  $\langle s^{out} \rangle(t) = \frac{N(N-1)+t}{N} = t/N + N - 1$ . The number of links in our problem is constant, similar to the number of nodes in Barabasi-Albert model B<sup>27</sup>, in which the authors applied the preferential attachment rule without adding new nodes. In our case we assume a preferential increase of directed weights. In the above-mentioned model B, the degree of node is proportional to  $t$ , i.e.  $k_i(t) \sim t$  for  $t > N$ . In our problem, mean-field-like analysis, i.e. substituting  $s_i^{out}$  with  $t/N + N - 1$ , and putting it to Eq.(2) leads to the equation:

$$w_{ij}(t) = \frac{N(N-1)+t}{N(N-1)+1} \approx \exp\left(\frac{t-1}{N(N-1)}\right), \quad (3)$$

which is correct for large values of  $N$  and  $t \ll N^2$ , where we used the initial condition  $w_{ij}(t=1) = 1$ . The process of weight increase is very slow in the case of large systems.

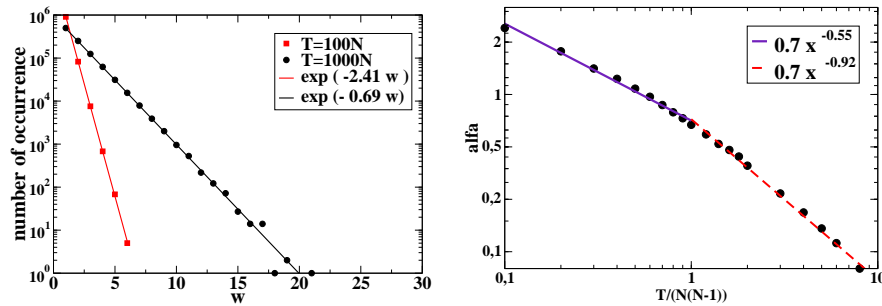


Fig. 1. The weight distribution is shown on the left, and the relationship between the parameter  $\alpha$  and  $\frac{T}{N(N-1)}$  is shown on the right (Model I).

Observing the weight distributions  $P(w)$  after  $T$  steps of simulation, we found that it follows an exponential behaviour with a characteristic exponent  $\alpha$  :

$$P(w) = A \exp(-\alpha w). \quad (4)$$

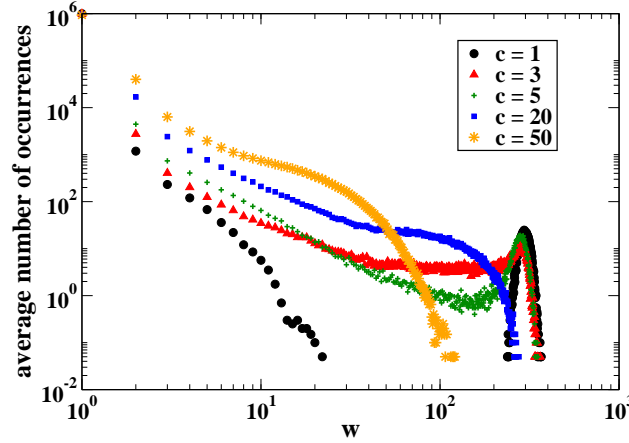


Fig. 2. Weight distribution for  $T = 300N$  and  $N = 1000$ ; 20 trials (Model II).

In Fig. 1, we present examples of weight distributions for different values of the time of simulation  $T$  for networks with  $N = 1000$ . A more detailed analysis of the dependence of exponent  $\alpha$  on the total time divided by the system size  $\frac{T}{N^2}$  is shown on the right side of Fig. 1. As one can see, the value of  $\alpha$  decreases with  $\frac{T}{N^2}$  following a power-law behaviour  $\alpha \sim (T/N^2)^{-\beta}$ , where two different regions of scaling can be distinguished.

### 3.3. Model II — with temporary memory

We develop **Model I** by introducing the concept of memory length. This approach allows us to consider a more realistic situation in which users forget very remote past events and make their decisions based only on the last transfer of messages. The updating procedure is almost identical to that in **Model I**, except that the selection probability is proportional to the number of past emotional messages transferred between nodes  $j$  and  $i$  in the last  $cN$  steps:

$$\frac{\partial w_{ij}(t)}{\partial t} = \frac{w_{ji}(t) - w_{ji}(t - cN)}{s_i^{in}(t) - s_i^{in}(t - cN)} \frac{1}{N}. \quad (5)$$

We observe how the weight distribution changes with parameters  $T$  and  $c$ . In the case of  $c < 3$  for small values of  $w_{ij}$  the distribution  $P(w_{ij})$  decreases while for larger  $w_{ij}$ , one can observe a Gaussian-like peak with the mean value equal to  $T/N$  (see Fig. 2). The value  $c = 3$  is a transition point; here, the two parts of the distribution merge, i.e. the Gaussian peak is absorbed by the decreasing part. For  $c > 5$ , there are once again traces of the Gaussian curve. For larger values of  $c$  (e.g.  $c = 20$  and  $c = 50$ ), the Gaussian part is completely invisible. To measure the evolution of weight distribution with changing memory length, we analyse the standard deviation defined as

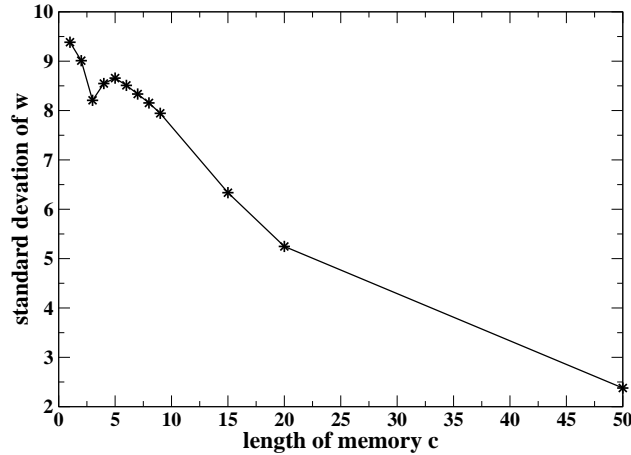


Fig. 3. Standard deviation as a function of the length of memory ( $T = 300N$ ,  $N = 1000$ ; 20 trials (Model II)).

$$\sigma = \sqrt{\frac{1}{N(N-1)} \sum_{ij} (w_{ij} - \langle w \rangle)^2}, \quad (6)$$

where  $\langle w \rangle = \frac{1}{N(N-1)} \sum_{ij} w_{ij}$  is the average weight of a link. The analysis confirms our previous observation that the lengths of memory  $c = 3$  and  $c = 5$  are transition points; in the standard deviation analysis, those points form local minimum and maximum, respectively (see Fig. 4).

For larger values of  $c$ , for example  $c = 50$ , we observed the following behaviour: for  $T = 100N$ , the character of the distribution is close to exponential (without the first point, which is dependent on the initial conditions), while for  $T = 450N$ , a fat tail with exponential cutoff is visible (see Fig. 4).

### 3.4. Model III — with secondary social sharing

Secondary social sharing is a phenomenon widely described in psychology. According to the famous researcher of this problem, Bernad Rimé<sup>36,37,38</sup>, *"First, it was predicted that being exposed to the social sharing of an emotion is emotion-inducing. Second, it was reasoned that if this holds true, then the listener should later engage in socially sharing with other persons the emotional narrative heard. Thus, a process of 'secondary social sharing' was predicted."* It was also proved by Rimé,<sup>37</sup> that the probability of social sharing phenomenon increases with the level of triggering emotions.

We try to adapt this phenomenon in order to extend our **Model II**. As the level of emotion is not considered in our models, we assumed that a secondary person

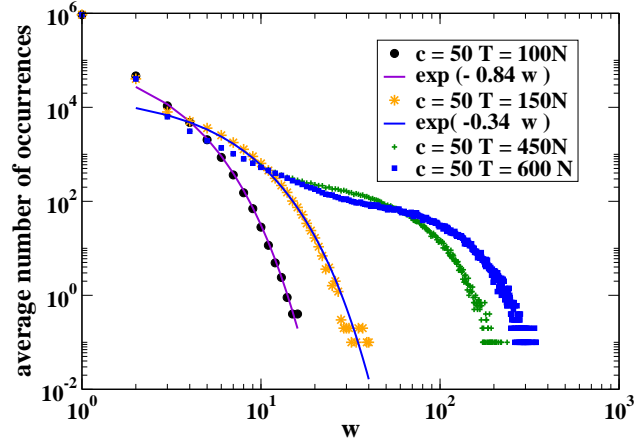


Fig. 4. Weight distribution, for  $N = 1000$  and  $T = 300N$ ; 20 trials (Model II).

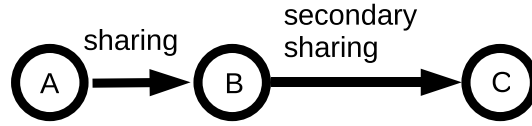


Fig. 5. Scheme of secondary social sharing process.

shares the emotional message with only one person (see Fig. 5). We randomly find a sender of a message and find the receiver using the rule from **Model II**. This receiver will then send a message to next person, using the same rule. The results obtained for this version of the model (see Fig. 6) are similar to Model II (see Fig. 2) with temporary memory; we also observe the decreasing part and Gaussian behaviour for the small value of  $c$ . A new feature of this distribution is an additional peak, for  $c = 5$ .

### 3.5. Model IV — chain letter

In this version, we assume that one user can send several emotional messages and one of their recipients becomes the sender of the next emotional message. The basic preferential rule with memory is still the same. We randomly select a user and randomly find the number of users to whom this agent sends a message. One of the recipients will send the message to a random number of users, which creates a chain of messages. The weighted distribution obtained from this version of the model is presented in Fig. 7. For  $c < 30$ , we can find a power-law scaling in the central regime (the first point is due to the influence of the initial condition and at the end one,

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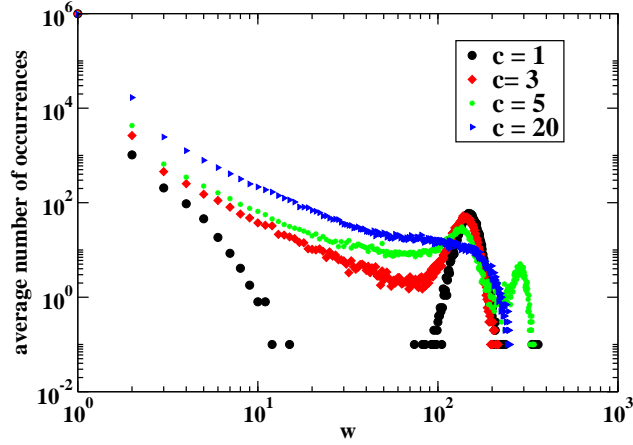


Fig. 6. Weights distribution, for  $N = 1000$  and  $T = 300N$ ; 10 trials (Model III).

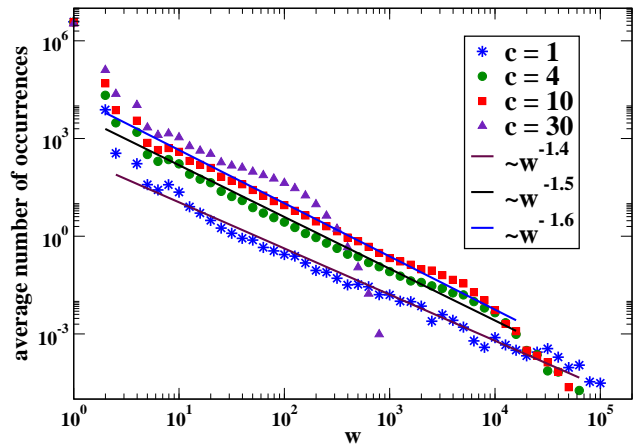


Fig. 7. Weights distribution, for  $N = 1000$  and  $T = 300N$ ; 10 trials (Model IV).

observes an exponential cutoff). This behaviour is qualitatively different from those in all previous cases. The introduction of the chain rule plays a pivotal role here. One can see the similarity to the random walk problem in a weighted network<sup>14</sup>.

#### 4. CONCLUSIONS

We analysed several models of communicating emotional messages in artificial e-communities that are described by directed weighted networks, where the weights/strengths of links correspond to the total number of messages sent from one agent to another. Model 0 assumes a random evolution of link strengths,  $w_{ij}(t)$ ,



and as a result, the distribution  $P(w)$  is Poissonian. In Model I, we assumed that messages possess an emotional character with a positive valence and that the willingness to send an affective message to a given person increases linearly with the number of messages already received from this person. As a result, we obtain an exponential behaviour of  $P(w)$ , where the characteristic exponent depends in a unique way on  $T/N^2$  ( $N$  is a network size and  $T$  is a network age). Introduction of a limited memory length  $c$  into Model II significantly changes the distributions  $P(w)$  that become the sum of the monotonically decaying part and a Gaussian peak. For larger values of the memory window  $c$ , the peak merges with the monotonic part. The effects of secondary social sharing phenomena were considered in Model III; however, the results were similar to those of Model II. In Model IV, we assumed a chain rule form of e-mail transmissions that resulted in the power-law behaviour of  $P(w)$ . The assumption of the finite memory length in Model II can be specific for some e-services where only limited temporal access to previously exchanged messages exists.

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